Intro to Options:

# More Greeks & Volatility

Vincent Vis
www.limitlong.com


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**After this module, you:**

- know all first-order Greeks
- understand their dynamics over strike and over time
- understand volatility, both implied and realised
- know a nice straddle price approximation


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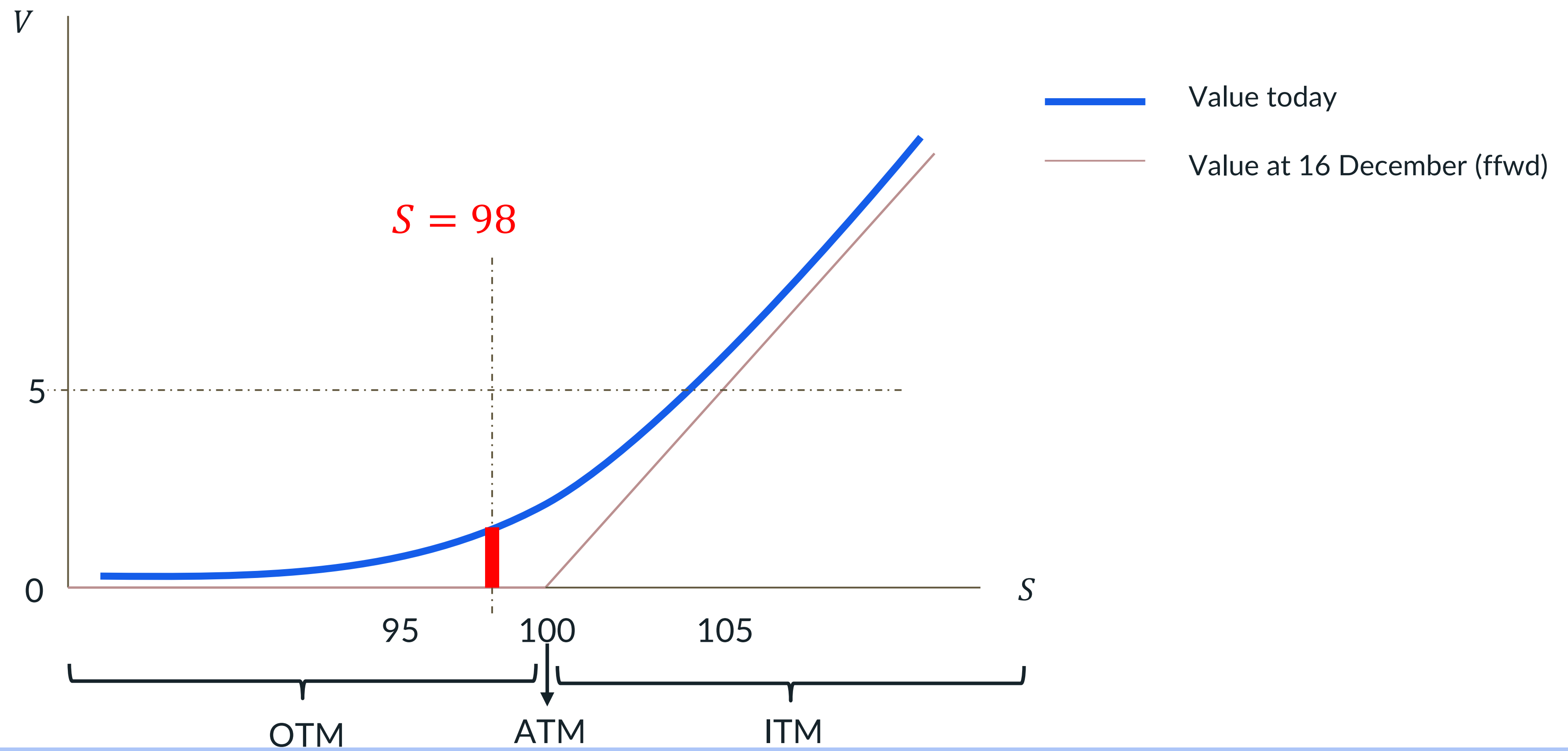
TODAY'S CONTENTS

# RECAP: THE VALUE OF AN OPTION DEPENDS ON:

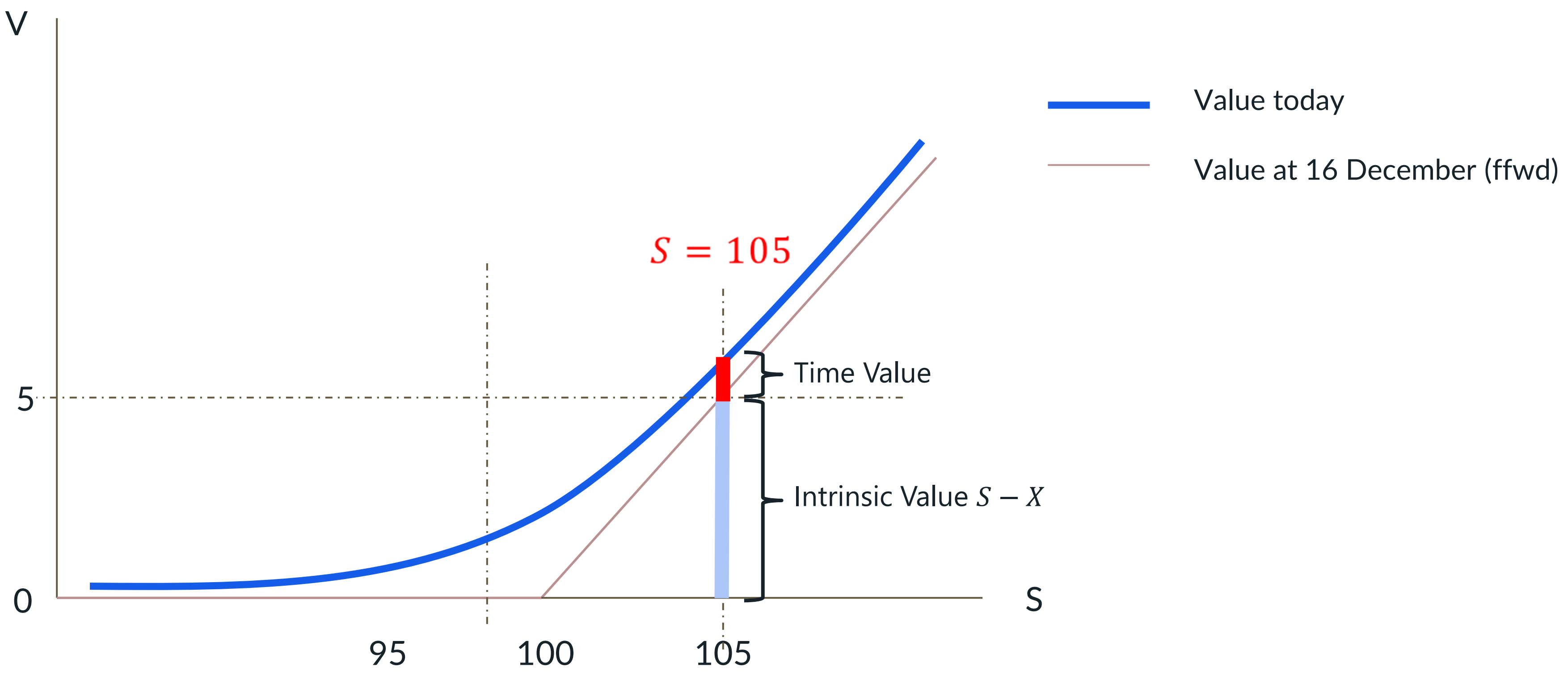
- Spot ($S$) and Strike ($X$) price of the underlying
- Time to expiry ($T$)
- Implied Volatility ($\sigma$)
- Interest Rate ($r$)
- Dividend ($D$)

Value $V := f(S, X, T, \sigma, r, D)$

# RECAP: HOW TO PRICE THE "SIE DEC 100 CALL" *TODAY* ?



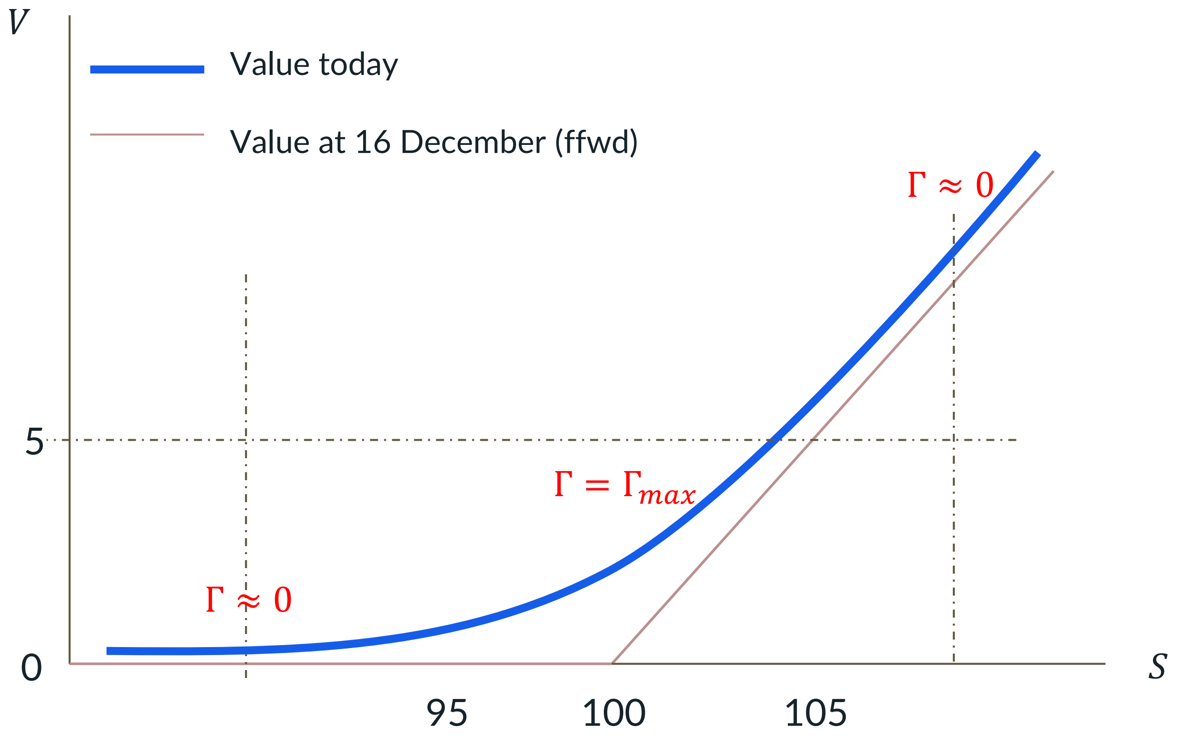
# RECAP: INTRINSIC VALUE VS TIME VALUE



# Sensitivity to each input are 'Greeks'

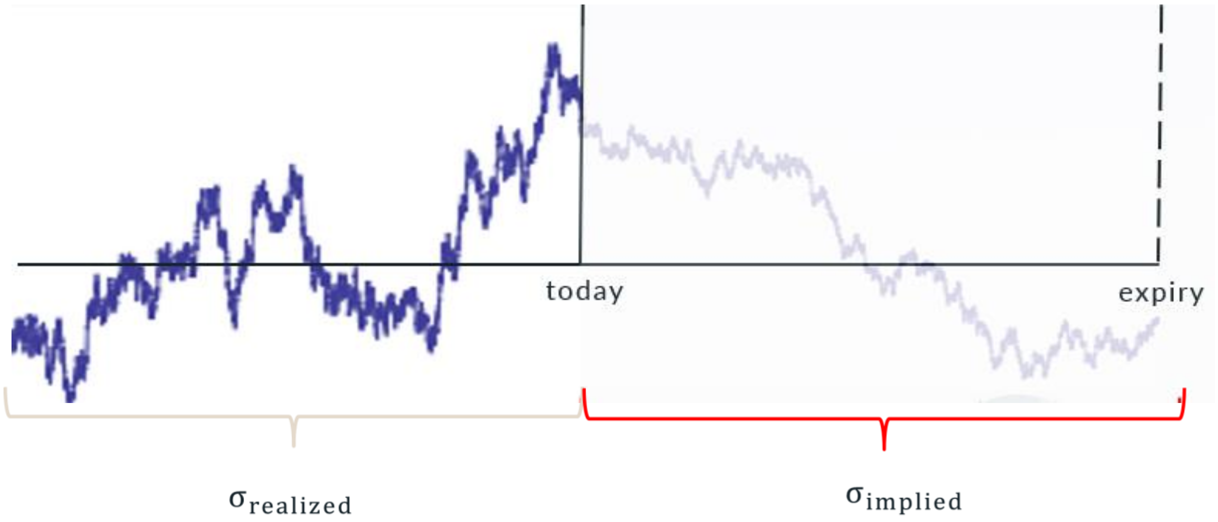
| ↑ | call | put | call Greek | put Greek |
|---|---|---|---|---|
| $S$ | ↑ | ↓ | $\Delta_c = \frac{\partial V_c}{\partial S} > 0$ | $\Delta_p = \frac{\partial V_p}{\partial S} < 0$ |
| $X$ | ↓ | ↑ | | |
| $t$ | ↑ | ↑ | | |
| σ | ↑ | ↑ | | |
| $r$ | ↑ | ↓ | | |
| $D$ | ↓ | ↑ | | |

# Recap: Gamma ($\Gamma$)



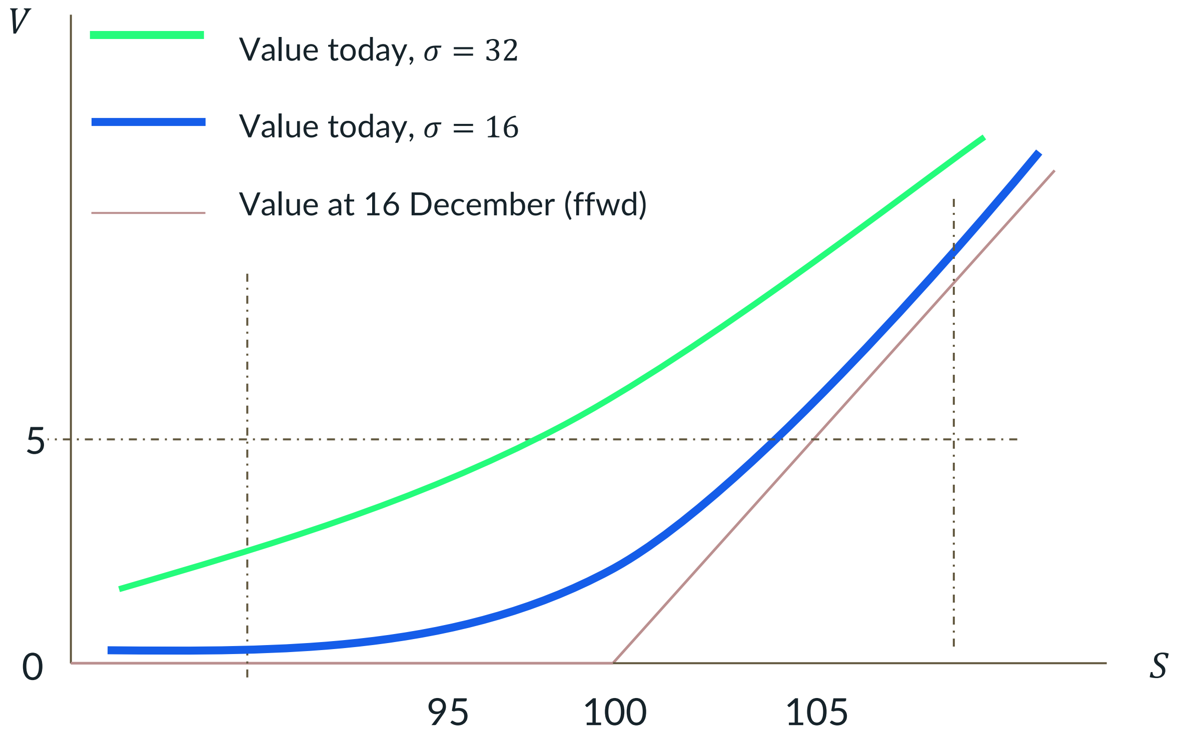

- change in delta per $1-change in the underlying: $\Delta = \frac{\partial \Delta}{\partial s}$
- can be a negative number when short options
- Biggest in magnitude ATM (vanilla options)

# VOLATILITY




- **Realized volatility** is a measure (standard deviation) over a time period (past performance)
- Rule of thumb: if $\sigma = 16$, the asset has an average daily move of 1%
- **Implied volatility** is the reverse engineered number from an option price (forward looking)

# GREEKS: Vega ($\nu$)




- Higher $\sigma_{\text{implied}}$ means higher option prices
- Vega is the sensitivity of the option's value for a one volpoint change: $\nu = \frac{\partial V}{\partial \sigma}$
- Vega decreases over time
- Vega is maximum for ATM options

PART II

## Sensitivity to each input are 'Greeks'

| ↑ | call | put | call Greek | put Greek |
|---|---|---|---|---|
| $S$ | ↑ | ↓ | $\Delta_c = \frac{\partial V_c}{\partial S} > 0$ | $\Delta_p = \frac{\partial V_p}{\partial S} < 0$ |
| $X$ | ↓ | ↑ | | |
| $t$ | ↑ | ↑ | | |
| $\sigma$ | ↑ | ↑ | $\nu_c = \frac{\partial V_c}{\partial \sigma} > 0$ | $\nu_p = \frac{\partial V_p}{\partial \sigma} > 0$ |
| $r$ | ↑ | ↓ | | |
| $D$ | ↓ | ↑ | | |

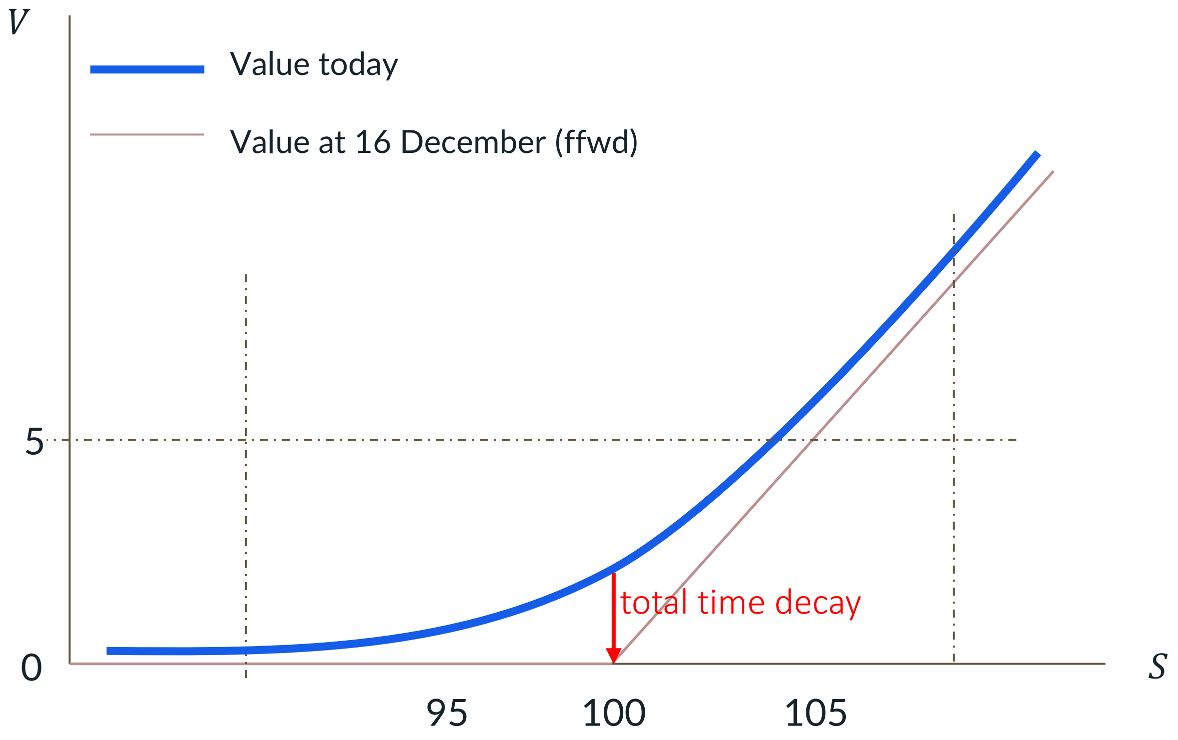
# EXERCISES (I)

- What is weighted vega?

- Long or short vega ($S = 100$)?
  - Short the 95/90 putspread
  - Long the 80/90 callspread
  - Long the 1M/3M 100 call calendar
  - Short the 1M/3M 100/90 put calendar

- The price of an ATM option is $ 3.30 and its vega is 10/ We raise implied volatility by 2.5 volpoints. Calculate the new price of the option.

# GREEKS: Theta ($\theta$)




- change in option value per one time unit (typically 24h):

$$-\theta = \frac{\partial V}{\partial t}$$

- Same direction for calls and puts
- Same characteristics as gamma: "*theta is the price of gamma*"

# Sensitivity to each input are 'Greeks'

| ↑ | call | put | call Greek | put Greek |
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| $t$ | ↑ | ↑ | $\theta_c = \frac{\partial V_c}{\partial t} > 0$ | $\theta_p = \frac{\partial V_p}{\partial t} > 0$ |
| σ | ↑ | ↑ | $\nu_c = \frac{\partial V_c}{\partial \sigma} > 0$ | $\nu_p = \frac{\partial V_p}{\partial \sigma} > 0$ |
| $r$ | ↑ | ↓ | | |
| $D$ | ↓ | ↑ | | |

# THETA AND P/L

- Formula for total profit & loss:

$$\text{Total PnL} = \text{Gamma PnL} + \text{ThetaPnL} + \text{other effects}$$

$$= \frac{1}{2} \cdot \Gamma \cdot \text{move}_{\$}^{2} - \theta \cdot (\Delta t)$$

- Movement required to make zero profit:

when $\sigma_{\text{implied}} = \sigma_{\text{realized}}$

- So when:

$$\text{move}_{\$} = \mathbb{E}(\text{Daily Move}) = \frac{\sigma \cdot S}{16}$$

# EXERCISES (II)

- Draw theta over towards expiry ($\theta$ versus $t$) for:
  - an ATM call option
  - a far OTM put option
- Do you pay or receive theta ($S = 100$)?
  - Short the 95/90 putspread
  - Long the 80/90 callspread
  - Long the 1M/3M 100 call calendar
  - Short the 1M/3M 100/90 put calendar
- True or False:
  "The sign of theta on a 90/80 1x2-ratio putspread is the same throughout its lifetime (assume $S = 90$, $\sigma_{\text{implied}}$ remains constant)."

PART II

# Rho ($\rho$) is the interest rate Greek

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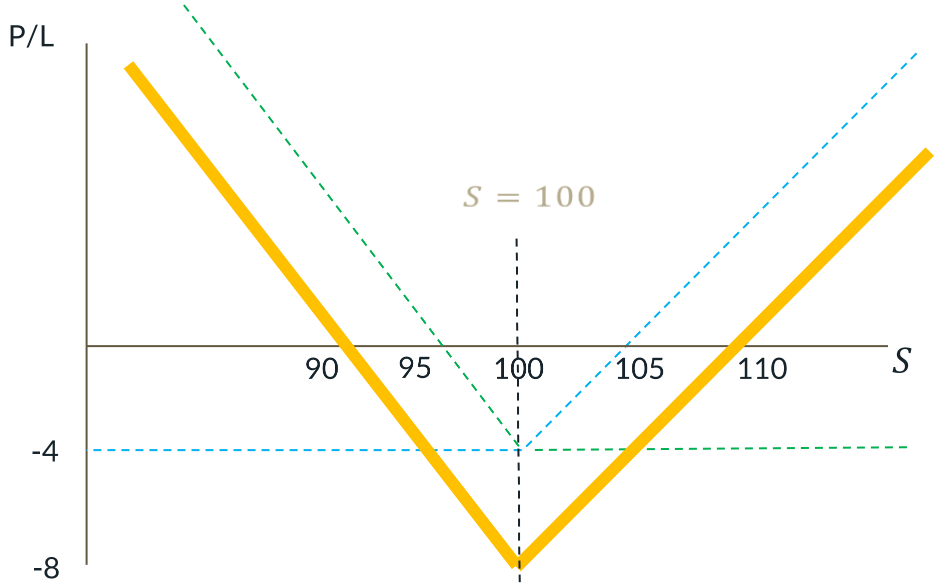
| σ | ↑ | ↑ | $\nu_c = \frac{\partial V_c}{\partial \sigma} > 0$ | $\nu_p = \frac{\partial V_p}{\partial \sigma} > 0$ |
| $r$ | ↑ | ↓ | $\rho_c = \frac{\partial V_c}{\partial r} > 0$ | $\rho_p = \frac{\partial V_p}{\partial r} < 0$ |
| $D$ | ↓ | ↑ | | |

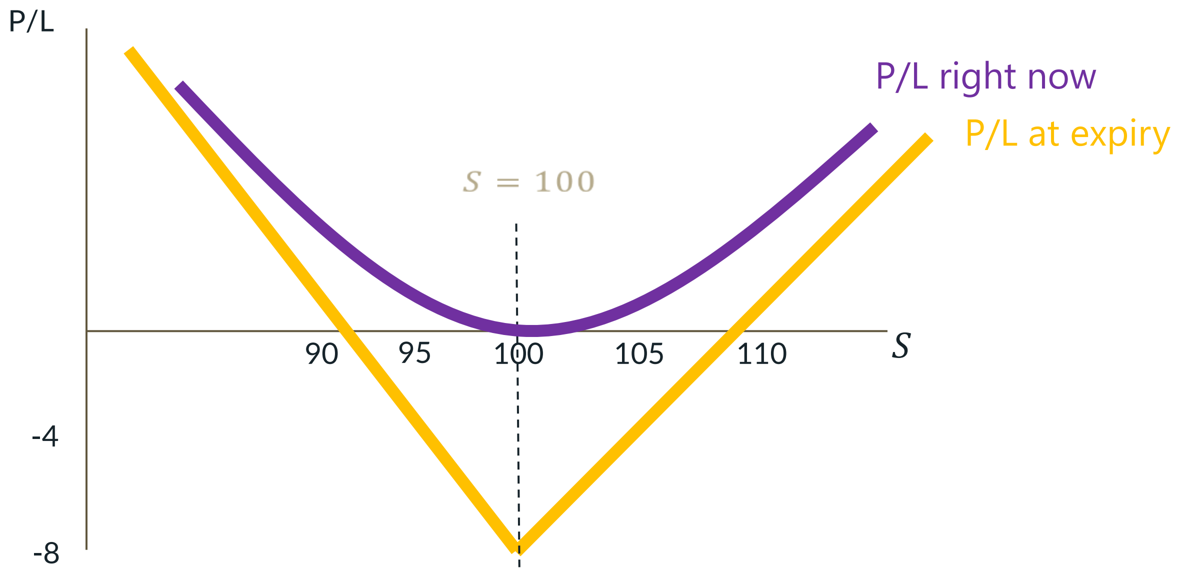
PART III

# LONG THE 100-STRADDLE (S = 100)




- Buy the 100 call and the 100 put, both for $4

# LONG THE 100-STRADDLE ($S = 100$)




- If $S$ stays at $S = 100$, how does time decay?

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PART III

# A USEFUL APPROXIMATION

Value $V := f(S, X, T, \sigma, r, D)$

- For ATM: $S = X$
- Assume $r = 0$
- Ignore higher order details
- Price of the ATM-straddle $Y_{ATM}$ :

$$Y_{ATM} \approx 0.8 \cdot S \cdot \sigma \cdot \sqrt{t}$$

# EXERCISE (III)

- Estimate the price of the S&P 3-months ATM-straddle. Sanity check the answer.
- Derive $\nu$ and $\theta$ from the approximation formula
- You bought an ATM-straddle at $\sigma_{\text{imp}} = 16$ and later you sold it when $\sigma_{\text{imp}} = 20$. Still you lost money. What happened?
- You bought a weekly ATM-straddle. If there will be one day where the underlying moves a lot, do you prefer it to be the first day or literally expiry day late afternoon? Or are you indifferent?

## THANKS SO MUCH FOR YOUR ATTENTION


vincent@limitlong.com

www.limitlong.com

www.linkedin.com/in/vincent-vis-6a8a8217/



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